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MAXIMUM HYPERGRAPHS WITHOUT REGULAR SUBGRAPHS

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Abstract

We show that an n-vertex hypergraph with no r-regular subgraphs has at most $2^{n-1}+r-2$ edges. We conjecture that if $n > r$, then every n-vertex hypergraph with no r-regular subgraphs having the maximum number of edges contains a full star, that is, 2^{n-1} distinct edges containing a given vertex. We prove this conjecture for $n \geq 425$. The condition that $n > r$ cannot be weakened.

Keywords: hypergraphs, set system, subgraph, regular graph.

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