## Math 412 HW5

Due Wednesday, February 21, 2024

Solve four of the next five problems.

1. Use Prim's Algorithm AND Kruskal's Algorithm to find minimum spanning trees in the weighted graph on the other side of this sheet (for each algorithm, show the sequence in which edges are added).

2. Calculate  $\tau(K_{2,4})$  using Matrix Tree Theorem. How many non-isomorphic spanning trees has  $K_{2,4}$ ?

3. Suppose that in the hypercube  $Q_k$ , each edge whose endpoints differ in coordinate *i* has weight  $2^i$ . Compute (with a proof) the minimum weight of a spanning tree in this  $Q_k$  with weighted edges.

4. Let G be a connected regular bipartite simple graph. Let G' be obtained from G by deleting one vertex from each of the two partite sets. Prove that G' has a perfect matching. Is the statement true when G is disconnected?

5. # 3.1.10 in the book.

Problems below review basic concepts and their ideas could be used in the tests.

WARMUP PROBLEMS: Section 2.3: # 2, 3. Section 3.1: # 1, 2, 3, 4, 5, 6, 7. Do not write these up!

OTHER INTERESTING PROBLEMS: Section 2.3: # 13, 14, 15. Section 3.1: # 8, 9, 16, 21, 24, 25, 26, 30, 31.

Do not write these up!

