02/20/24

Math 412 **HW6**

Due Wednesday, March 06, 2024

Solve four of the next five problems.

1. # 3.1.28 in the book.

2. Let D be a digraph. Prove that there exist pairwise disjoint cycles in D such that each vertex of D lies in exactly one of the cycles if and only if $|N^+(S)| \geq |S|$ for all $S \subseteq V(D)$. (Note that $N^+(S)$ may intersect or contain S.) (Hint: The trick used to prove Petersen's Theorem on 2-factors may help.)

3. Given the preference list below, determine the stable matchings resulting from the Proposal Algorithm run (a) with men proposing and (b) with women proposing.

Men $[u, v, w, x, y, z]$	Women $[a, b, c, d, e, f]$
$u: \ a > b > d > c > f > e$	$a: \ z > x > y > u > v > w$
$v: \ a > b > c > f > e > d$	$b: \ y > z > w > x > v > u$
$w: \ c > b > d > a > f > e$	$c: \ v > x > w > y > u > z$
$x: \ c > a > d > b > e > f$	$d: \ w > y > u > x > z > v$
$y: \ c > d > a > b > f > e$	$e: \ u > v > x > w > y > z$
$z: \ d > e > f > c > b > a$	$f: \ u > w > x > v > z > y$

Write down the results of each round of proposals for both parts, (a) and (b).

4. Construct a connected 3-regular simple graph G such that the size of the maximum matching in G is at most $\frac{|V(G)|}{2} - 2$.

5. Let G be a 9-regular connected graph that remains connected after deleting any 7 edges. Prove that G has a perfect matching.

Problems below review basic concepts and their ideas could be used in the tests.

WARMUP PROBLEMS: Section 3.2: # 3. Section 3.3: # 1, 2, 6, 7. Do not write these up!

OTHER INTERESTING PROBLEMS: Section 3.2: # 12, 13, 14. Section 3.3: # 8, 9, 10, 15, 16, 22, 24, 25, 26. Do not write these up!