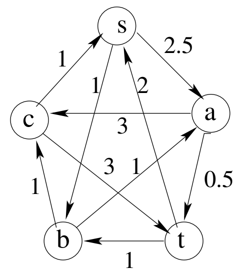
Math 412 **HW8**

Due Wednesday, April 10, 2024

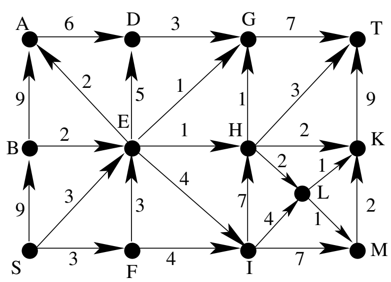
Solve four of the next five problems.

1. Let $k \geq 3$. Prove that for every $n \geq k+1$, every k-connected n-vertex graph G, and every disjoint vertex sets S and T in G with $|T| = 3$ and $|S| = k-3$, there is a cycle that contains T and is disjoint from S. (Hint: Use the Fan Lemma.) Give an example of a 2-connected graph and some 3 vertices in this graph that do not belong to a common cycle.

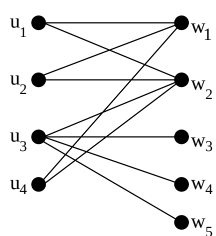
2. Let G be the network with the flow drawn below. Write the flow as a linear combination of flows along cycles, s,t-paths and t,s-paths.



3. In the network below find an S,T-cut of minimum capacity. Prove that it has the minimum capacity.



4. Using maximum flows (solution without flows does not count!), find a maximum matching in the bipartite graph below. Prove that the matching is optimal. Find a smallest vertex cover.



5. Let (G, ϕ) be a 3-connected simple plane graph, let n_i denote the number of vertices of degree i in G, and let f_j denote the number of faces of degree j in (G, ϕ). Prove that

$$\sum (4-i)n_i + \sum (4-j)f_j = 8. \qquad \text{(Hint: Use Euler's Formula.)}$$

Problems below review basic concepts and their ideas could be used in the tests.
WARMUP PROBLEMS: Section 4.2: # 5. Section 4.3: # 1. Do not write these up!
OTHER INTERESTING PROBLEMS: Section 4.2: 12, 22, 28. Section 4.3: # 5, 7, 13. Do not write these up!