04/15/24

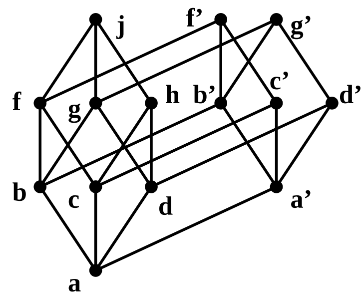
Math 412 **HW 9**

Due Friday, April 26, 2024

Solve four of the next five problems.

1. Let (G, ϕ) be a connected 4-regular plane simple graph in which every vertex lies on two (opposite) faces of length 5 and on two (opposite) faces of length 3. Use Euler's formula to find the number of edges and the number of faces of (G, ϕ)

2. Let Q_4^* denote the graph obtained from the 4-dimensional cube Q_4 by deleting two adjacent vertices (see the picture below). Determine whether Q_4^* is planar or not and prove your answer.



3. A graph H is the *square* of a graph G if $V(H) = V(G)$ and xy is an edge in H if and only if $x \neq y$ and the distance between x and y in G is at most two. Prove that for $n \geq 5$, the square, C_n^2, of the cycle C_n is planar if and only if n is even.

4. For a chess piece Q, *the Q-graph* is the graph whose vertices are the squares of the chess board and the two squares are adjacent if Q can move from one of them to the other in one move. Find the chromatic number of the Q-graph when Q is (a) the king, (b) a rook, (c) a bishop, (d) a knight.

5. Prove or disprove: For every n and every n-vertex graph G, $\chi(G) \leq 3\omega(G) + \frac{3n}{\alpha(G)} + 3$.

Problems below review basic concepts and their ideas could be used in the tests.

WARMUP PROBLEMS: Section 6.1: # 1, 3, 4, 7, 8, 9, 10. Section 6.2: # 1, 2. Section 5.1: # 1, 4, 7, 8, 12, 14, 15. Section 5.2: # 1, 2, 3. Do not write these up!

OTHER INTERESTING PROBLEMS: Section 6.1: # 18, 25, 27, 29, 30. Section 6.2: # 5, 7, 8, 11. Section 5.1: # 33, 38, 39, 41. Section 5.2: # 6, 8, 9, 15. Do not write these up!