Please, write here your name: ______________________________

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6

February 27, 2020

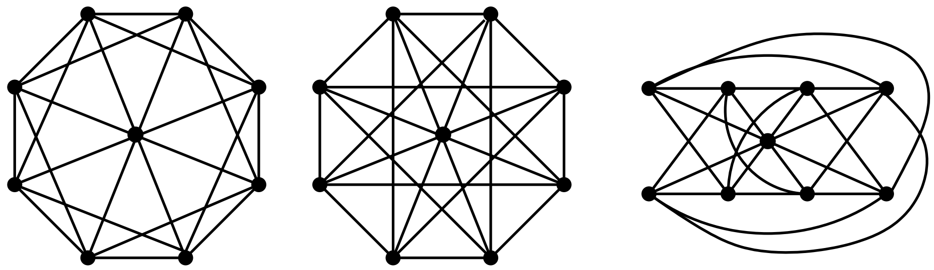
MATH412 **Exam 1**

1. Which of the following are graphic sequences? Provide a construction or a proof of impossibility for each:
(a) $(4, 4, 3, 3, 3, 2, 2, 2, 1, 1)$, (b) $(5, 5, 5, 4, 2, 1, 1, 1)$, (c) $(5, 5, 5, 3, 2, 2, 1, 1)$.

2. Let G be a graph with at least two vertices. Prove or disprove:
(a) Deleting a vertex of degree $\Delta(G)$ cannot increase the average degree.
(b) Deleting a vertex of degree $\delta(G)$ cannot decrease the average degree.

3. Using the Prüfer correspondence, for $n \geq 8$, count the number of trees with vertex set $[n]$ that have maximum degree exactly 4 and exactly 5 leaves. (Hint: Start from determining which vertex degrees should be in a tree with 5 leaves and a vertex of degree 4.)

4. Determine which graphs below are isomorphic and which are not. (Hint: it is simpler to consider the complements of the graphs.)



(a) (b) (c)

5. Prove the following part of the characterization theorem for trees. Prove that an n-vertex graph G is a tree if and only if G is connected and has $n-1$ edges. You can use Lemma 2.1 which claims that each tree has a leaf and that if we delete a leaf from a connected graph then the resulting graph is connected.

6. (a) State the Matrix Tree Theorem (with all details);
(b) Define and draw Petersen's graph;
(c) May a disconnected graph have minimum degree 3? Please, either explain why not or give an example of such a graph.