01/17/24

Math 412 **HW1**

Due Wednesday, January 24, 2024 by 4pm.

Instructions: Students taking the course for three credit hours (undergraduates, most graduate students) should choose four of the following five problems to solve and turn in. Graduate students taking the course for four credits should solve all five. Problems that use the word "describe" or "determine" require proof for all claims.

1. Consider the following four families of simple graphs: $A = \{\text{graphs with 5 or 6 edges}\}$, $B = \{\text{complete bipartite graphs with both parts nonempty}\}$, $C = \{\text{cycles}\}$, $D = \{\text{complements of a 3-regular simple graph}\}$. For each pair of these families, determine all graphs that belong to both families in the pair.

2. Determine which pairs of graphs in Fig. 1 below are isomorphic and which are not.

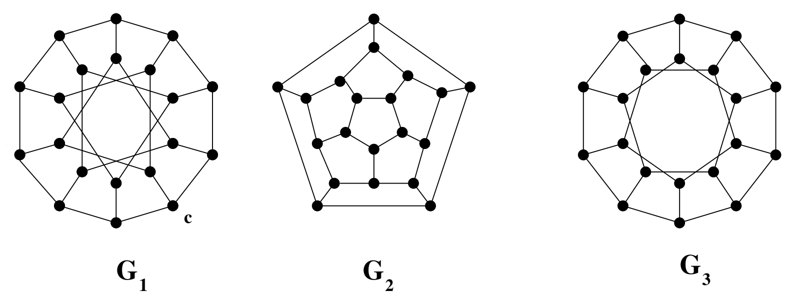

Figure 1.

3. For every of the four properties below, determine (up to isomorphism) all connected simple graphs G satisfying this property:
(a) G has 4 vertices and no cycles;
(b) G has 4 vertices and exactly one cycle;
(c) G is 3-regular and has 5 vertices,
(d) G is 4-regular and has 7 vertices.

4. For each of the classes $\mathcal{C}_i$ below determine the smallest n for which $\mathcal{C}_i$ contains nonisomorphic n-vertex graphs with the same degree sequence:
(a) $\mathcal{C}_1 = \{\text{all graphs}\}$, (b) $\mathcal{C}_2 = \{\text{loopless graphs}\}$, (c) $\mathcal{C}_3 = \{\text{simple graphs}\}$.
(Hint: You can use the list in Example 1.1.31 of the book to give a lower bound in Case (c).)

5. Let G be a simple graph with vertex set $\{v_1, \ldots, v_n\}$ and adjacency matrix $A = \{a_{i,j}\}_{i,j=1}^{n}$.
(a) For $1 \le i < j \le n$, what does the entry in position (i, j) of A^2 say on vertices v_i and v_j of G?
(b) Prove that the ith diagonal entry of the matrix A^2 is the degree of vertex v_i in G.

Problems below review basic concepts and their ideas could be used in the tests/quizzes.

WARMUP PROBLEMS: Section 1.1: # 2, 4, 5, 7, 9. Do not write these up!

OTHER INTERESTING PROBLEMS: Section 1.1: # 13, 16, 18, 20, 21, 22, 23, 26, 29, 37. Do not write these up!