Math 412   **HW2**

Due Wednesday, January 31, 2024

Solve four of the next five problems.

1. Given an integer $k \geq 2$, let $G(k)$ be the subgraph of the cube Q_{2k} induced by the vertices in which the number of ones is either $k - 1$ or k. Compute the number of vertices, the number of edges, and the girth (the length of a shortest cycle) of $G(k)$. What are the degrees of the vertices in $G(k)$?

2. Prove or disprove:
(a) Every connected graph G has a closed walk that traverses each edge of G exactly four times;
(b) Every connected graph G has a closed walk that traverses each edge of G exactly three times.

3. Find the number of maximum independent sets in the Petersen Graph P.

4. Prove that for every positive integer k, every connected graph with exactly $2k$ vertices of odd degree and arbitrarily many vertices of even degree decomposes into k trails. Does this remain true without the connectedness hypothesis?

5. # 1.2.28 in the book.

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Problems below review basic concepts and their ideas could be used in the tests.
WARMUP PROBLEMS: Section 1.2: # 1, 4, 5, 8, 9, 10, 11. Do not write these up!
OTHER INTERESTING PROBLEMS: Section 1.2: # 14, 18, 20, 23, 41.
Section 1.3: # 17. Do not write these up!