02/06/24

Math 412 **HW4**

Due Wednesday, February 14, 2024

Solve four of the next five problems.

1. **Using a generalized de Bruijn graph with alphabet** $\{0, 1, 2\}$ **instead of** $\{0, 1\}$**,** find a cyclic arrangement of 27 digits $0, 1$, and 2 such that all 27 strings of 3 consecutive digits are distinct. (Hint: The graph has 9 vertices.)

2. Let $n \geq 3$ and G be an n-vertex graph. Prove that if at least 3 induced subgraphs of G obtained by deleting one vertex are acyclic (i.e., have no cycles), then G has at most one cycle.

3. Given $x \in V(G)$, let $s(x) = \sum_{v \in V(G)} d(x, v)$. The *barycenter* of G is the set its vertices at which $s(x)$ is minimized.

a) Prove that the barycenter of a tree is a single vertex or two adjacent vertices. (Hint: Study $s(u) - s(v)$ when $uv \in E(G)$.)

b) Give an example of a tree in which the distance between the center and the barycenter is at least 3.

4. Let $n \geq 2$. Find the number of spanning trees in the graph obtained from the complete graph K_n by adding a non-loop edge.

5. Using the Prüfer correspondence, for $n \geq 10$, count the number of trees with vertex set $[n]$ that have maximum degree 3 and exactly six leaves. (Hint: Start from finding out how many vertices of degree 3 such a tree has.)

Problems below review basic concepts and their ideas could be used in the tests.

WARMUP PROBLEMS: Section 2.1: # 1, 2, 4, 6, 13. Section 2.3: # 1, 2, 3. Do not write these up!

OTHER INTERESTING PROBLEMS: Section 1.4: # 36, 37. Section 2.1: # 15, 19, 27, 29, 31, 44, 52, 53. Section 2.3: # 6, 14, 15.

Do not write these up!