Representations, isomorphism

Lecture 2

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The **degree** of a vertex $v \in V(G)$ will be denoted by $d(v)$ or $d_G(v)$ (when G is not clear from context). The **maximum degree** of G is $\Delta(G) = \max\{d(v) \mid v \in V(G)\}$.
Similarly the **minimum degree** of G is
$\delta(G) = \min\{d(v) \mid v \in V(G)\}$.

We say G is k**-regular** if every vertex has degree k.

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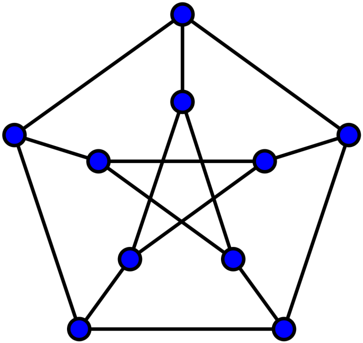

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The vertex set of the k-dimensional cube Q_k is $V_k = \{(a_1, \ldots, a_k) : a_i \in \{0, 1\}\}$ and two vectors in V_k are adjacent iff they differ in exactly one coordinate.

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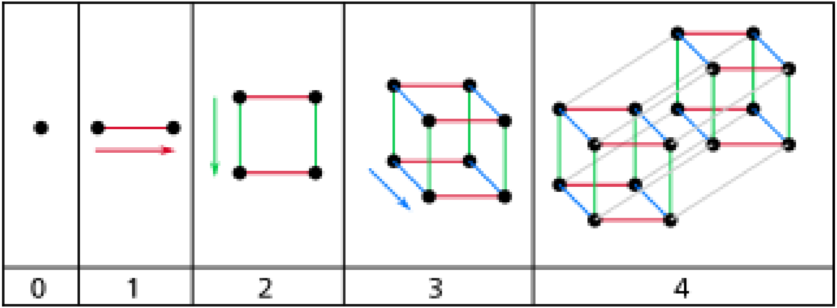

(c) Adjacency matrices. Given a loopless graph G with vertex set $\{v_1, \ldots, v_n\}$, the adjacency matrix $A(G)$ of G is the $n \times n$ matrix $\{a_{i,j}\}_{1 \leq i,j \leq n}$ where $a_{i,j}$ is equal to the number of edges with endpoints v_i and v_j.

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(e) Lists of neighbors. Given a simple graph G with vertex set $\{v_1, \ldots, v_n\}$, for every v_i the list of its neighbors is given.

Graph isomorphism

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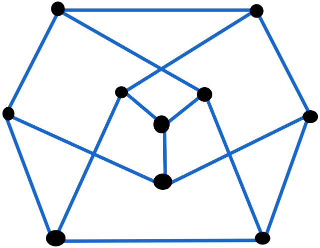
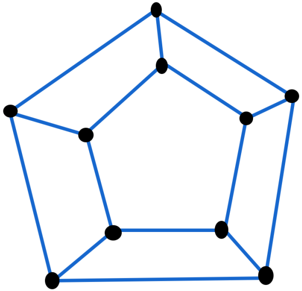
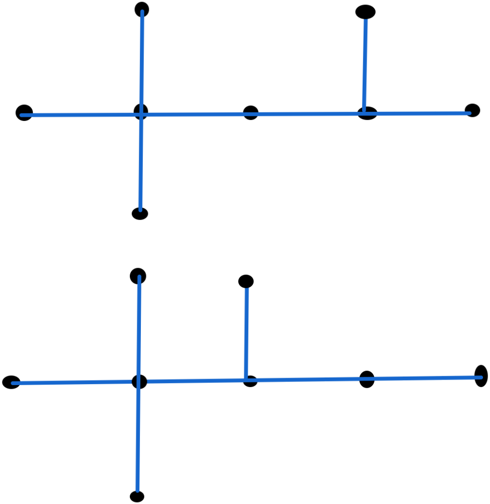
Two graphs G and H are isomorphic if there is an isomorphism from G to H.

Isomorphism, Example 1:





Isomorphism, Example 2:



Walks

A **walk** in a graph G is a list $v_0, e_1, v_1, e_2, v_2, \ldots, e_\ell, v_\ell$ of vertices v_i and edges e_i such that for each $1 \leq i \leq \ell$, the endpoints of e_i are v_{i-1} and v_i.

If the first vertex of a walk is u and the last vertex on the walk is v, we call this a u, v-walk. When G is a simple graph, we also may specify a walk by simply listing the vertices, since it is unambiguous which edge is traversed in each step.

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A u, v-**trail** is a u, v-walk with no repeated edges (but vertices may repeat). If $u \neq v$, a u, v-**path** is a u, v-walk with no repeated vertices.
(You should convince yourself that the subgraph definition of a path matches up with the walk definition of a path).
If $u = v$, then we call a u, v-walk or trail **closed**. The **length** of a walk, trail or path is the number of edges traversed.