MATH 582, FALL 2021 – PROBLEM SET 3

Do five of the six problems below. Due Wednesday, October 27.

1. Find necessary and sufficient conditions for a graphic sequence to be the degree sequence of a connected (simple) graph.

2. For positive integers n and k such that $n \geq k+2$ and $n-k$ is odd, show that the degree sequence consisting of $k-1$ numbers $n-1$ and $n-k+1$ numbers k is graphic, but has no k-connected realization. What about $(k-1)$-connected realizations?

3. Prove that the vertex set of every 5-degenerate graph G with maximum degree 6 can be partitioned into sets V_1, V_2, V_3 and V_4 so that V_1 and V_2 are independent, and every component of $G(V_3)$ and of $G(V_4)$ is a path (possibly, with just one vertex).

4. Prove that the degree sequence of a graph with at least four edges is edge-reconstructible. (Hint: Start from the number of vertices of maximum degree.) Use this to prove that
(a) every graph with at least four edges and degrees of all vertices of the same parity is edge-reconstructible;
(b) the Edge-Reconstruction Conjecture holds for graphs with exactly four edges.

5. Let G be an n-vertex graph with a non-trivial automorphism group and $e(G) \geq \log_2(n!) \geq 4$. Prove that G is edge-reconstructible.

6. Prove that
(a) Every n-vertex graph ($n \geq 3$) such that at least $n-2$ cards in the deck are disconnected is reconstructible;
(b) Every n-vertex graph ($n \geq 5$) with maximum degree $n-1$ is edge-reconstructible.