UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
DEPARTMENT OF MATHEMATICS
Course Description — FALL 2021

MATH 582

STRUCTURE OF GRAPHS

Section F1: 2pm – 2:50pm MWF, 445 Altgeld Hall.
Instructor: A. Kostochka, 234 Illini Hall, 265-8037, kostochk@illinois.edu.
Office hours: tentatively MWF 3:10–4:10 and by appoinment.
Web page: https://kostochk.web.illinois.edu/math582.

TEXT: D. B. West, The Art of Combinatorics, Volume II: Structure of Graphs. For some topics, instructor's supplements will be provided.

TOPICS: This is a companion course to Math 581 — Extremal Graph Theory. The two courses are independent. *Structure of Graphs* includes topics are drawn from the following (not all will be covered).

Elementary Structural Concepts — structural and enumerative topics involving trees and related graphs, degree sequences, embeddings of graphs in product graphs. Graph packings and equitable colorings.

The reconstruction problem — is G reconstructible from the deck of subgraphs obtained by deleting a single vertex? ... a single edge?

Connectivity — min-max relations for connectivity and branchings, structure of k-connected graphs.

Cycles — Hamiltonian cycles and circumference in graphs and digraphs.

Topological Graph Theory — embeddings on surfaces (without edge crossings), characterizations and properties of graphs embeddable in the plane (separator theorems, proof of Kuratowski's Theorem, Schnyder labelings), measures of non-planarity, voltage graphs and chromatic number of surfaces. Using discharging for coloring problems on surfaces.

Joins and flows — the language of conservative weightings for finding maximal joins and minimum T-joins, cycle covers and nowhere-zero flows.

Graph Minors — treewidth and the minor order, some discussion of Robertson-Seymour Theorem (every minor-closed family of graphs has infinitely many minimal forbidden minors), forbidden and forced minors.

COURSE REQUIREMENTS: There will be 5 problem sets, each requiring 5 out of 6 problems for 50 points total; no exam. The instructor also may add up to 30 points for active participation in class. The solutions need to be typed (apart from pictures). Roughly speaking, 220 points suffices for an A, 180 for a B. Discussions between students about problems can help understanding. Collaborations should be acknowledged, and submitted homework should be written individually in your own words.

PREREQUISITES: Familiarity with elementary graph theory. Either of Math 580 and Math 412 provides sufficient preparation. Interested students with no graph theory background may browse a basic text in advance, such as Diestel, Graph Theory, or the Math 412 text: West, Introduction to Graph Theory (Prentice Hall, 2001, first seven chapters).

SEE THE OTHER SIDE:

Following University policy, all students are required to engage in appropriate behavior to protect the health and safety of the community. Students are also required to follow the campus COVID-19 protocols.

Students who feel ill must not come to class. In addition, students who test positive for COVID-19 or have had an exposure that requires testing and/or quarantine must not attend class. The University will provide information to the instructor, in a manner that complies with privacy laws, about students in these latter categories. These students are judged to have excused absences for the class period and should contact the instructor via email about making up the work.

Students who fail to abide by these rules will first be asked to comply; if they refuse, they will be required to leave the classroom immediately.

All students, faculty, staff, and visitors are required to wear face coverings in classrooms and university spaces. This is in accordance with CDC guidance and University policy and expected in this class.